

# The extended Betz–Lanchester limit

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## Abstract

A proposal for an extended formulation of the power coefficient of a wind turbine is presented. This new formulation is a generalization of the Betz–Lanchester expression for the power coefficient as function of the axial deceleration of the wind speed provoked by the wind turbine in operation. The extended power coefficient takes into account the benefits of the power produced and the cost associated to the production of this energy.

By the simple model proposed is evidenced that the purely energetic optimum operation condition giving rise to the Betz–Lanchester limit (maximum energy produced) does not coincide with the global optimum operational condition (maximum benefit generated) if cost of energy and degradation of the wind turbine during operation is considered.

The new extended power coefficient is a general parameter useful to define global optimum operation conditions for wind turbines, considering not only the energy production but also the maintenance cost and the economic cost associated to the life reduction of the machine.

## Nomenclature

$a$	axial induced factor
$C_P$	power coefficient
$C_P^E$	extended performance coefficient
$C_T$	thrust coefficient
$\bar{C}_T$	averaged thrust coefficient
$K$	cost ratio
$k$	proportionality constant between $\Delta l$ and $L_E$
$k_e$	ratio of the economic benefit obtained from transformed energy over energy produced at constant power during a period of time.
$k_l$	ratio of the total investment (proportional to total life time of the machine) over the accumulated load state of the machine
$k_{me}$	ratio of the operation/maintenance cost over energy produced at constant power during a period of time
$k_2$	proportionality constant between $L_E$ and $U_\infty^2$
$k_3$	proportionality constant between $L_E$ and total thrust
$L_E$	global equivalent load representative of the fatigue loading of the wind turbine
$L_T$	total damage accumulated by the wind turbine
$N_H$	number of hours per year
$N_Y$	number of years
$R$	radio of wind turbine rotor
$S$	rotor surface
$t$	time during which $L_E$ is acting on the machine
$u$	wind speed at the rotor plane
$U_{AVE}$	annual averaged wind speed
$U_\infty$	undisturbed wind speed
$V_0$	cut-in wind speed
$V_1$	cut-out wind speed
$\delta_1$	net benefit parameter
$\delta_2$	net cost parameter
$\Delta I_c$	part of total investment consumed per unit of time, when the wind turbine is loaded at a wind speed $U_\infty$ with a thrust coefficient $C_T$
$I_c$	total investment cost
$\Delta l$	actual life reduction due to a global equivalent load $L_E$
$\rho$	air density

when the system decelerates the wind speed down to  $2/3$  the wind speed of the undisturbed flow upstream.

The concept of Betz–Lanchester optimisation is purely energetic, and does not establish any consequence nor on the structural behaviour of the system neither on the life time reduction associated to this energetically optimum condition.

Since the pioneer concept of wind turbine optimisation provided by Betz, the techniques for optimising the design and operation of wind turbines have been intensely developed as the size, cost and number of wind turbines have increased.

Additionally, the Betz–Lanchester limit establishes a theoretical limit for the performance of a wind turbine of normal configuration (no diffuser, no tip vane).

However, a more useful approach to the optimisation problem of a wind turbine requires a quite more complex mathematical treatment [2], the complexity being related mainly to the following aspects:

First, a wind turbine is an aeroelastic system, which suffers aging process during operation. The system is quite complex to be modelled from a physical-mathematical point of view and presents a highly complex interaction with the environment (meteorological and electrical network conditions [5]).

Second, the environment is too complex to be modelled due to the difficulty to reproduce the long period climatic phenomena and the short period turbulence. A similar problem appears when dealing with the simulation of the electrical network environment or the demand.

Third, the optimisation criteria are frequently in opposition. Sometimes the situations that lead to an optimum operation from the energetic point of view (maximum energy transformed) gives rise to an extremely high damage accumulation and consequently life reduction. The optimisation process must be a multi-criteria one, maximizing parameters related with the total cost of energy.

Fourth, the mathematical techniques needed to solve the previous problems are complex and highly costly in terms of computing requirements.

The complexity of the problem has lead to the establishment of partial solutions since the former approach by Betz. The models of wind turbine aerodynamics based on Blade Element Theory combined with Momentum Theory allowed former optimisations of the rotor geometry (number of blades, taper, torsion) of a wind turbine, oriented to maximize the energetic performance, [8,13]. An excellent review of these techniques can be found in Ref. [3].

At the beginning of the eighties, an important effort was devoted by the scientific and technical community to incorporate structural aspects in the optimisation process. This development was supported on a parallel enhancement of the knowledge of the environmental parameters that influenced the overall behaviour of the wind turbine.

Several parameters which characterise the atmospheric flow and that affect the behaviour of the wind turbine have been identified [9,15].

Therefore, at that time, it was assumed that an optimisation model should incorporate proper simulation algorithms of the atmospheric flow, Mann [14], and that the model of the wind turbine aerodynamics must be complete enough, incorporating corrections to the Blade Element–Momentum Theory, new Vortex Theories, [20], Snel and Schepers [18] or CFD simulations, [10,11].

Additionally the structural model of the wind turbine must be accurate enough, describing the system as an elastic solid with enough degrees of freedom, [11].

All the previous developments were included in several aeroelastic tools, able to simulate the aeroelastic behaviour of a wind turbine operating in a turbulent flow, [12,17].

The development of the previous tools opened the way to the application of optimisation tools defining general optimisation objectives, which considered multi-criterion aspects leading to a maximisation of the total benefit, [2–4,6,7,19].

Apart from the remarkable works, previously cited, optimisation of wind turbine present some incomplete aspects.

First, wind speed is described by means of a Weibull distribution, a turbulence model and a simple shear law. No time arrow is considered in the wind simulation (only a statistical weighting of the phenomena) making incomplete the simulation of time evolution phenomena as aging of components due to fatigue or loss of energy due to disconnection/connection process of wind turbines (hysteresis process).

Second, the economic model of maintenance cost are not accurate enough. Maintenance cost is one of the driving factors determining the total cost of wind turbines. Additionally they are influenced by the design and the operation policy during the wind turbine life, therefore, the proper simulation of their evolution is crucial in a proper global optimisation process.

## 2. The extended Betz–Lanchester limit

As it has been described the current optimisation process of wind turbines is quite complex. This complexity makes difficult to obtain easy consequences on how to operate a wind turbine if the total cost of energy must be optimised.

The method proposed here aims to demonstrate, in a simple way, that energy optimum criteria and structural optimum criteria can be combined in a Betz–Lanchester like manner, to obtain simple optimum conditions for a simplified energy-structural description of a wind turbine.

As it is quite well known, the simple Momentum Theory analysis of an ideal rotor operating in ideal conditions gives rise to the following expressions for the power coefficient and thrust coefficient:

$$C_P = \frac{P}{\frac{1}{2}\rho S U_\infty^3} = 4a(1 - a)^2 \quad (1)$$

$$C_T = \frac{T}{\frac{1}{2}\rho S U_\infty^2} = 4a(1 - a) \quad (2)$$

where  $P$  is the power produced by the wind turbine,  $T$  is the total thrust,  $C_P$  is the Power Coefficient,  $C_T$  is the Thrust Coefficient,  $\rho$  is the air density,  $S$  is the rotor surface,  $U_\infty$  is the undisturbed air speed and  $a$  is the axial induced factor, being  $a = u/U_\infty$ , with  $u$  the induce velocity at the rotor plane.

Expressions (1) and (2) are strictly valid for one-dimensional, steady and uniform conditions and ideal rotors, [13], although they represent quite well the averaged tendency of real rotors.

Additionally, even though the previous conditions are satisfied, there is a range of the parameter  $a$ , where the expressions (1) and (2) provide coherent values for  $C_P$  and  $C_T$ , being this interval  $a \in (0,0.4)$ .

The main hypotheses of this work are: First the economic benefit obtained from transformed energy is proportional through a constant  $k_e$  to the power produced. Second the operation/maintenance cost are calculated through the constant  $k_{me}$  as a value proportional to the power produced. Finally the total investment (proportional to the total life time of the machine) is reduced proportionally, through a constant,  $k_l$ , to the overall load state of the machine, represented by the wind turbine thrust.

Therefore, the proposed Extended Power Performance Coefficient is defined as:

$$C_P^E = \frac{\text{Economical net benefits}}{\text{Economical quantification of flow power}} = \frac{(k_e - k_{me}) \frac{1}{2} \rho S U_\infty^3 C_P - k_l \frac{1}{2} \rho S U_\infty^2 C_T}{k_e \frac{1}{2} \rho S U_\infty^3} \quad (3)$$

giving rise to:

$$C_P^E = 4a(1 - a) \left[ \left( 1 - \frac{k_{me}}{k_e} \right) (1 - a) - \frac{k_l}{k_e} \frac{1}{U_\infty} \right] \quad (4)$$

To consider the life time reduction proportional to an overall loading state, represented here by total thrust, may be understood as an extremely simplifying hypothesis. However, it is valid if a linear relation between life reduction and fatigue loads is considered acceptable (it is not of course necessarily true, but for the purposes of this work, it is considered sufficient) as:

$$\Delta l = -k_l L_E t \quad (5)$$

where  $\Delta l$  is the actual life reduction,  $L_E$  is a global equivalent load representative of the fatigue loading of the wind turbine,  $t$  is the time during which  $L_E$  is acting on the machine, and  $k$  is the proportionality constant. If units of  $l$  are  $[T]$ ,  $k$  units are  $(M^{-1} L^{-1} T^2)$

In Pierik, Cuerva et al. [15], the equivalent loads referred to in Eq. (5) are modelled as a quadratic function of standard deviation of wind speed. Additionally, if a linear relation between standard deviation of wind speed and mean wind speed in a certain period of time is assumed, Pierik, Cuerva et al. [15],  $L_E$  can be written:

$$L_E \approx k_2 U_\infty^2 \quad (6)$$

In the previously cited reference, also a linear term in  $U_\infty$  appears. Here this term has been removed, for sake of simplicity and considering that  $U_\infty$  is large enough (at least larger than cut-in wind speed).

Additionally, expression (2) indicates that, for constant values of induced axial induced parameter,  $a$ , the total thrust of a wind turbine presents a quadratic dependency with  $U_\infty$ . Considering this fact along with Eqs. (5) and (6) it is possible to write the actual life reduction,  $\Delta l$ , as:

$$\Delta l \approx -k_3 \rho S U_\infty^2 C_T \quad (7)$$

Expression (7) also indicates that the total thrust has been considered as representative of the total loading state of the machine. Of course this is not exactly true, since specific weak overloaded components normally exist, but is valid to illustrate, in a first approach, the economic effect of considering life reduction due to load consideration.

Apart from fatigue loading, isolated high loading episodes lead to life reduction. However, these are not considered in order to reduce the complexity of the formulation.

If total operational life of a wind turbine is related to a total investment cost,  $I_c$ , which is maximum at the beginning of the operation and 0 at the end, it is possible to write that:

$$\Delta I_c \approx -k \frac{1}{2} \rho S U_\infty^2 C_T \quad (8)$$

where  $\Delta I_c$  [ $\$ T^{-1}$ ] is the part of total investment consumed per unit of time, from now on referenced as structural cost, when the wind turbine is loaded at a wind speed  $U_\infty$  with a thrust coefficient  $C_T$ .  $k_1$  has units of [ $\$ M^{-1} L^{-1} T^1$ ] where  $\$$  is the handled economic unit.

### 3. Optimum operation of a wind turbine based on the extended Betz–Lanchester limit

Expression (4) is rewritten as:

$$C_P^E = 4\delta_1 a(1-a) \left[ (1-a) - \frac{\delta_2}{\delta_1 U_\infty} \right] \quad (9)$$

where  $\delta_1 = 1 - (k_{me}/k_e)$  and  $\delta_2 = (k_1/k_e)[LT^{-1}]$ . Renaming  $K = (\delta_2/\delta_1 U_\infty)$ , expression (9) becomes:

$$C_P^E = 4\delta_1 a(1-a)[(1-a) - K] \quad (10)$$

$K$  is named from now on the cost ratio.

For a constant  $C_P$  value wind turbine (a machine controlled to operate in an optimum value of parameter  $a$ ) the second term of Eq. (10) grows linearly with the inverse of wind speed  $U_\infty$ , meaning that for lower wind speeds, the Extended Performance Coefficient is reduced, since, the total investment is consumed while producing a small amount of energy. Also, should be noted that there is a threshold wind speed, that gives a 0 value for  $C_P^E$ . Again if constant  $a$  operation is assumed:

$$U_\infty \Big|_{C_P^E=0} = \frac{\delta_2}{\delta_1(1-a)} \quad (11)$$

Wind speeds larger than  $U_\infty \Big|_{C_P^E=0}$  give rise to positive values of  $C_P^E$  and therefore to worthwhile operation of the wind turbine.

As shown in Eq. (10), for a given operation status, constant  $a$ , and constant wind speed  $U_\infty$ , the Extended Performance Coefficient grows as  $\delta_1$  grows and  $\delta_2$  decreases (resulting in a decreasing  $K$  for a given wind speed) meaning that both the cost of operation and the investment reduction due to a certain loading level reduce down compared to the prize paid for energy selling.

Derivation of expression (10) with regard to  $a$ , and equating to 0 gives rise to the equation that allows to obtain the optimum axial induced factor as follows:

$$\frac{\partial C_P^E}{\partial a} = 4\delta_1[(1 - 4a + 3a^2) + K(2a - 1)] = 0 \quad (12)$$

It must be noted that if  $\delta_1=1$  and  $\delta_2=0$  ( $K=0$ ) which means that nor structural cost neither operation cost are considered, expression (12) leads to Betz–Lanchester formulation.

Eq. (12) is solved for axial induced factor  $a$ , to give the optimum axial induced factor,  $a_{OP}$ , which leads to the maximum value of the Extended Performance Coefficient. Fig. 1 shows the value of optimum axial induced factor,  $a_{OP}$ , divided by the value predicted by Betz–Lanchester ( $a=1/3$ ) versus parameter  $K$ . The value predicted by Betz–Lanchester is recovered for  $K=0$ . As it has been mentioned,  $K \rightarrow 0$ , for a given wind speed, means a negligible operation/maintenance and structural cost compared to economic benefit associated to energy selling ( $\delta_2 \ll \delta_1$ ). The solution given by Betz–Lanchester ( $3a_{OP}=1$ ) is a particular case of this situation corresponding to  $\delta_2=0$  and  $\delta_1=1$  ( $K \equiv 0$ ).

Considering Fig. 1, if for certain wind conditions parameter  $K$  results 0.2, to obtain a maximum extended performance coefficient, the wind turbine should be operated (settings of pitch and rotational speed) to produce and averaged induced axial factor 12% lesser than the optimum value prognosticated by Betz–Lanchester. That operating condition, for instance would be achieved by reducing the pitch angle, giving rise to a

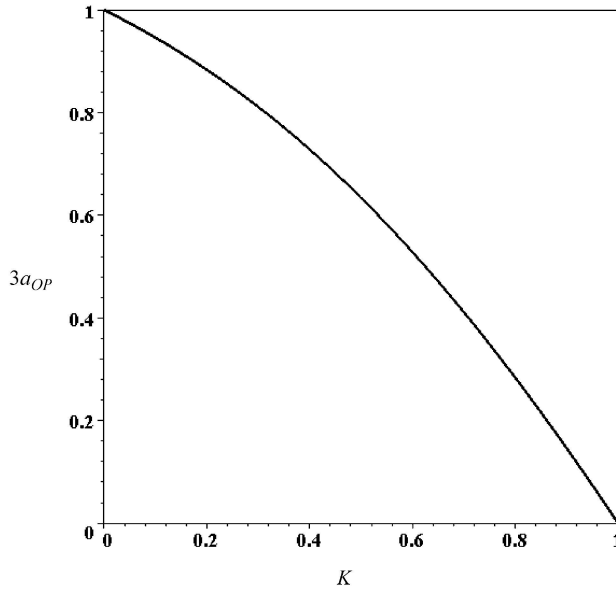


Fig. 1. Optimum induced axial factor,  $a_{OP}$ , leading to maximum extended performance coefficient,  $C_P^E$ .  $K$ , relative cost parameter. The values of the optimum induced axial factor are presented scaled with the maximum value provided by the Betz–Lanchester theory.

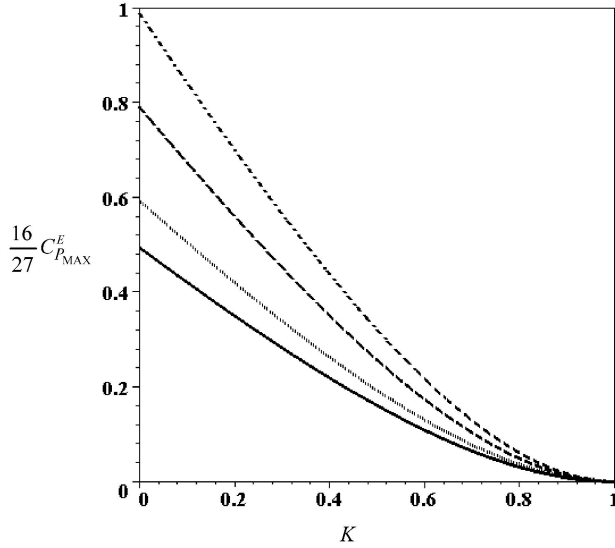


Fig. 2. Maximum extended performance coefficient,  $C_P^E$  versus parameter  $K$ , and  $\delta_1$  as parameter. Dotted-dashed line:  $\delta_1=1$ , dashed line:  $\delta_1=0.8$ , dotted line:  $\delta_1=0.6$ , solid line:  $\delta_1=0.5$ .

reduction of the energy extracted but also to a reduction of the loads generated of the structure that globally would give rise to a more optimum situation that the obtained by a purely energetic analysis ( $a=1/3$ ).

Fig. 2 shows the values of  $C_P^E$  divided by the value predicted by Betz–Lanchester Theory ( $C_{P_{MAX}}^E = 16/27 \approx 0.6$ ) as a function of  $K$  and  $\delta_1$  as a parameter. It can be seen that Betz–Lanchester value ( $C_{P_{MAX}}^E/0.6 \sim 1$ ) is again recovered for  $\delta_1=1$  and  $K=0$  (neither operation/maintenance nor structural cost). The value of maximum Extended Performance Coefficient is reduced as  $\delta_1$  reduces (higher operation/maintenance costs, lower selling price of energy) and  $\delta_2$  grows (higher structural costs) giving rise to decreasing  $K$ .

For expected values of parameter  $K$ , (see point 4)  $K \in (0.05, 0.2)$ , values for  $C_{P_{MAX}}^E$  in the order of 40–80% of the Betz–Lanchester values are expected.

#### 4. Estimation of the cost parameters $\delta_1$ , $\delta_2$ and $K$

As an example, parameters  $\delta_1$  and  $\delta_2$  (and therefore  $K$ ) can be estimated under some simplifying assumptions.  $k_e$  has been defined as the selling price of one kWh. This value can vary from one region to another, and of course with time, but can be estimated in general.  $k_e \approx 0.06$  euro/kWh will be assumed in this example. The operation/maintenance cost constant,  $k_{me}$ , is estimated 0.05–0.20 times  $k_e$  depending on the period of operation of the wind turbine (initial years to final years) Rademarkers, Braam et al. [16].



Finally the constant  $k_1$  represents the total investment reduction (reduction in operation life quantified economically, previously defined as structural costs) during one hour producing a thrust of one kN.  $k_1$  has, therefore, dimensions of euro/kNh.  $k_1$  can be estimated (once it is assumed that thrust can represent the overall loading state of the wind turbine) for one specific site and wind turbine. For a given wind speed probability distribution  $f(U_\infty)$ , then the total load accumulated during the total life of a wind turbine can be estimated in the following way:

The total amount of damage accumulated by the machine is expressed in kNh as:

$$L_T = 10^{-3} N_H N_Y \int_{V_0}^{V_1} \frac{1}{2} \rho S U_\infty^2 C_T(a) f(U_\infty) dU_\infty \approx \frac{10^{-3}}{2} N_H N_Y \bar{C}_T \rho S \int_{V_0}^{V_1} U_\infty^2 f(U_\infty) dU_\infty \quad (13)$$

where  $L_T$  is the total damage accumulated by the wind turbine,  $N_H$  is the total number of hours of a year,  $N_Y$  is the number of years of operation,  $S$  is the rotor disk area,  $V_0$  and  $V_1$  are the cut in and cut off wind speeds, and  $\bar{C}_T$  is a representative averaged value for the time history of the thrust coefficient.

In expression (13) the damage accumulated during periods with the wind turbine disconnected have been neglected.

The parameter  $\delta_2$  can be estimated as:

$$k_1 = \frac{I_c}{L_T} \quad (14)$$

A practical case has been run for the following typical values:

Rated power: 1500 kW.

Total installation cost,  $I_c$ : 901500 €.

Rotor radius,  $R$ : 70 m.

Average thrust coefficient,  $\bar{C}_T$ : 0.3.

Ratio of the economic benefit obtained from transformed energy over energy produced at constant power during a period of time (1 h),  $k_e$ : 0.06 €/kWh).

$k_{me}/k_e$ , 0.006;

$N_H$ : 8760 hours/years.

$N_Y$ : 20 years.

Considering the previous data, the constant  $\delta_1$  results

$$\delta_1 = 1 - \frac{k_{me}}{k_e} = 0.9 \quad (15)$$

From expression (13) and (14) and the definition of parameter and  $\delta_2$  given previously, parameter  $\delta_2$  results a function of the annual average wind speed,  $U_{AVE}$ .

Fig. 3 shows the dependency of parameter  $\delta_2$  with the annual average wind speed,  $U_{AVE}$ . As it is shown, as  $U_{AVE}$  increases, meaning that the site is more energetic, the parameter  $\delta_2$  decreases indicating that the cost associated to life reduction is proportionally lesser relatively to the economic income.

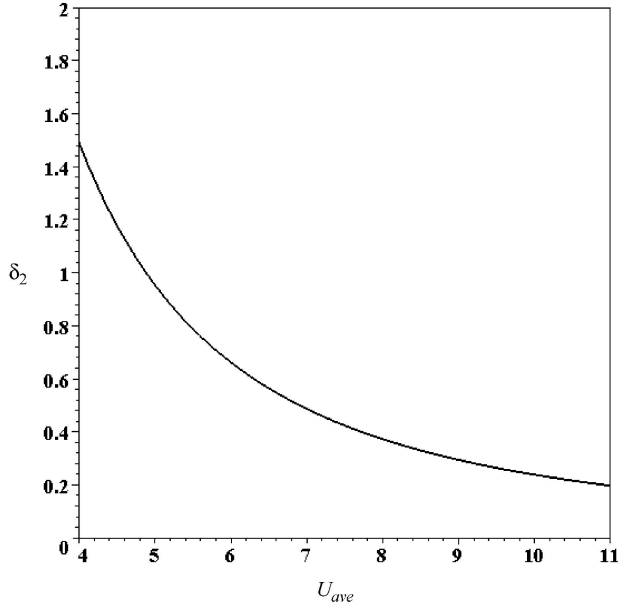


Fig. 3. Dependency of parameter  $\delta_2$  versus the annual average wind speed,  $U_{AVE}$ . Calculated for a Rated power of 1500 kW, Total installation cost,  $I_C$ : 901500 €, Rotor radius,  $R$ : 70 m, Average thrust coefficient,  $\bar{C}_T$ : 0.3, Ratio of the economic benefit obtained from transformed energy over energy produced at constant power during a period of time (1 h),  $k_e$ : 0.06 €/kWh,  $k_{me}/k_e$ : 0.006. Number of hours per year:  $N_H$ : 8760, number of years of operation:  $N_Y$ : 20.

If a typical place with an annual average wind speed,  $U_{AVE}=7.5$  m/s at hub height is chosen, then from Fig. 3 a value  $\delta_2=0.5$  is estimated. Considering the result from expression (15) for  $\delta_1$ , the resulting value for parameter  $K$  is  $K=5/(9U_\infty)$ . For this situation the variation of the Extended Performance Coefficient versus axial induced factor,  $a$ , is presented using wind speed,  $U_\infty$  as parameter in Fig. 4.

In Fig. 4, the solid, intermediate thickness line, represents the Betz–Lanchester formulation of the power coefficient ( $C_P^E$  for  $\delta_1=1$ ,  $K=0$ ). The family of thin lines represents the evolution of  $C_P^E$  versus  $a$ , for increasing values of wind speed,  $U_\infty$ . As can be seen, increasing values of wind speed,  $U_\infty$ , lead to increasing values of the extended performance coefficient,  $C_P^E$ . This fact indicates that, for the case studied, the increment of economic benefits associated to a higher amount energy transformed (and therefore energy selling) is higher than the total cost associated to maintenance, operation and structural cost associated to this wind condition.

Additionally, the solid thick line shows the loci of  $C_{P_{MAX}}^E$  values. The axial induced factor,  $a$ , giving rise these optimum extended performance coefficient values increases as wind speed,  $U_\infty$ , does. This result is a particularization of the general behavior showed in Fig. 1. In this case, for a given  $\delta_1$  and  $\delta_2$ , increasing wind speeds imply decreasing  $K$  values.

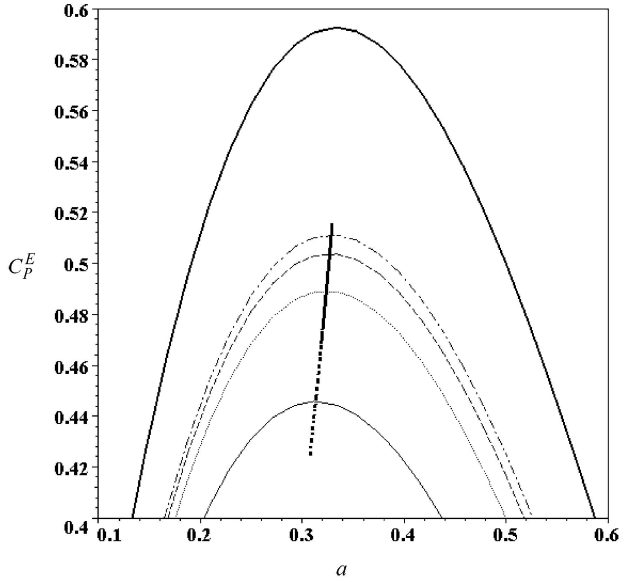


Fig. 4. Extended performance coefficient,  $C_P^E$ , versus axial induced factor,  $a$ .  $U_\infty$  as parameter. Solid intermediate thickness line: Betz–Lanchester curve (Power coefficient given by expression (1)), dotted-dashed line:  $U_\infty = 15$  m/s., dashed line:  $U_\infty = 15$  m/s., dotted line:  $U_\infty = 10$  m/s., solid thin line:  $U_\infty = 5$  m/s. The line of maxima has been marked by the solid thick line indicating the variation of axial induced factor for different wind speeds. The parameters are valued as follows:  $\delta_1 = 0.9$ ,  $\delta_2 = 0.5 \text{ ms}^{-1}$  [ $K = 5/(9U_\infty)$ ].

Considering the previous reasoning, the operation settings of the wind turbine (pitch and rotor wind speed) should be changed so that the proper axial induced factor is achieved (given by the loci of  $C_{P_{\text{MAX}}}^E$  values in Fig. 4), instead of the initially targeted  $a = 1/3$ .

## 5. Conclusions

A model for an extended performance coefficient including global net benefit criteria is presented. The model allows to consider the influence of the benefit of energy selling, as well as the cost of producing energy (operating and maintenance costs) and the cost associated to the life reduction of the wind turbine due to operation (structural costs).

The model identifies global optimum situations which are function of wind speed (once the economic parameters are fixed) indicating that the pure energetic optimum does not necessarily coincide with the global optimum operational condition.

The classic solution for optimum operational condition is recovered (as expected) when no cost associated to the structural life reduction is considered.

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